ANSWERS

Cross sectional properties

2.1.1 a) For a coordinate system with its origin at C:
Z = (200; 262.5) mm
b) $I_{zz} = 39.1 \times 10^8$ mm^4
$I_{yy} = 25.3 \times 10^8$ mm^4
$I_{yz} = I_{zy} = 0$
c) Using Mohr's Circle method or tensortransformation rules:
$I_{zz} = I_{yy} = 32.2 \times 10^8$ mm^4
$I_{yz} = I_{zy} = 6.9 \times 10^8$ mm^4
d) $I_{z'z'} = 149.4 \times 10^8$ mm^4
$I_{y'y'} = 89.3 \times 10^8$ mm^4
$I_{y'z'} = I_{z'y'} = 84 \times 10^8$ mm^4

2.1.2 A) (cross section a)
a) For a coordinate system with its origin at the bottom left corner:
Z = (-30; -30) mm
b) See book Volume 2: Stresses, Strains, Displacements,
§3.2.4 **Example 4**

$$I_{zz} = \tfrac{1}{12} bh^3$$

$$I_{yy} = \tfrac{1}{12} b^3 h + \tfrac{1}{12}\frac{bh^3}{\tan^2\alpha} \quad with: \tan\alpha = 2$$

$$I_{yz} = \tfrac{1}{12}\frac{bh^3}{\tan\alpha}$$

$I_{zz} = 54 \times 10^4$ mm^4
$I_{yy} = 27 \times 10^4$ mm^4
$I_{yz} = I_{zy} = 27 \times 10^4$ mm^4

c) Using Mohr's Circle or transformation rules: $\alpha_1 = -31{,}7°$; $\alpha_2 = 238{,}3°$
d) $I_1 = 70.7 \times 10^4$ mm^4
$I_2 = 10.3 \times 10^4$ mm^4

B) (cross section b)
a) For a coordinate system with its origin at the bottom left corner:
Z = (-30; -20) mm
b) Using Mohr's Circle method:
$I_{zz} = 54.0 \times 10^4$ mm^4
$I_{yy} = 121.5 \times 10^4$ mm^4
$I_{yz} = I_{zy} = -40.5 \times 10^4$ mm^4
c) $\alpha_1 = -25.1°$
$\alpha_2 = 244.9°$
d) $I_1 = 140.5 \times 10^4$ mm^4
$I_2 = 35.0 \times 10^4$ mm^4

C) (cross section c)
a) For a coordinate system with its origin at the bottom left corner:
Z = (-40; -20) mm
b) Using Mohr's Circle method:
$I_{zz} = 54 \times 10^4$ mm^4
$I_{yy} = 94.4 \times 10^4$ mm^4
$I_{yz} = I_{zy} = -13.5 \times 10^4$ mm^4
c) $\alpha_1 = -16{,}9°$
$\alpha_2 = 253.1°$
d) $I_1 = 98.5 \times 10^4$ mm^4
$I_2 = 49.9 \times 10^4$ mm^4

2.1.3 b) $I_{z"z"} = 1/4\ bh^3$

2.1.4 A) (left cross section)
a) For a coordinate system with its origin at the top left corner:
Z = (-25; 11.667) mm
b) Using Mohr's Circle method:
$I_{zz} = 10.75 \times 10^4$ mm^4
$I_{yy} = 22.75 \times 10^4$ mm^4
$I_{yz} = I_{zy} = 6 \times 10^4$ mm^4
c) $\alpha_1 = 22.5°$
$\alpha_2 = 292.5°$
d) $I_1 = 25.2 \times 10^4$ mm^4
$I_2 = 8.3 \times 10^4$ mm^4

B) (right cross section)
a) For a coordinate system with its origin at the top left corner:
Z = (-50; 40) mm
b) Using Mohr's Circle method:
$I_{zz} = 263.3 \times 10^4$ mm^4
$I_{yy} = 141.7 \times 10^4$ mm^4
$I_{yz} = I_{zy} = -131.6 \times 10^4$ mm^4
c) $\alpha_1 = 32.6°$
$\alpha_2 = 302.6°$
d) $I_1 = 347.5 \times 10^4$ mm^4
$I_2 = 57.5 \times 10^4$ mm^4

2.1.5 A) (left cross section)
a) For a coordinate system with its origin at the top left corner:
Z = (-2/3a; a)
b) Using Mohr's Circle method:
$I_{zz} = 4.667ta^3$
$I_{yy} = 2.667ta^3$
$I_{yz} = I_{zy} = 0$
c) $\alpha_1 = 0°$

$\alpha_2 = 270°$
d) $I_1 = 4.667ta^3$
$I_2 = 2.667ta^3$
e) at $\alpha = -45°$, $I_{yz} = I_{zy} = ta^3$
at $\alpha = 225°$, $I_{yz} = I_{zy} = -ta^3$

B) (right cross section)
a) For a coordinate system with its origin at the top left corner:
Z = (-a; a)
b) Using Mohr's Circle method:
$I_{zz} = 4.667ta^3$
$I_{yy} = 1.333ta^3$
$I_{yz} = I_{zy} = -2ta^3$
c) $\alpha_1 = 25.1°$
$\alpha_2 = 295.1°$
d) $I_1 = 5.6ta^3$
$I_2 = 0{,}4ta^3$
e) at $\alpha = 45°$, $I_{yz} = I_{zy} = -2.6ta^3$
at $\alpha = 315°$, $I_{yz} = I_{zy} = 2.6ta^3$

2.1.6 a) For a coordinate system with its origin at the bottom right corner:
Z = (100; -150) mm
b) Using Mohr's Circle method:
$I_{zz} = 210 \times 10^6$ mm^4
$I_{yy} = 120 \times 10^6$ mm^4
$I_{yz} = I_{zy} = 80 \times 10^6$ mm^4
c) $\alpha_1 = -30.3°$
$\alpha_2 = 239.7°$
$I_1 = 257 \times 10^6$ mm^4
$I_2 = 73.2 \times 10^6$ mm^4

Normal stresses in case of bending

```
> restart;
> My:=(1/2)*F*L;
> Mz:=-(1/2)*sqrt(3)*F*L;
> Iyy:=2*(1/12)*t*a^3;
> Izz:=(1/12)*t*a^3+2*t*a*((1/2)*a)^2;
> kappay:=My/(E*Iyy);
> kappaz:=Mz/(E*Izz);
> sigma:=E*(kappay*y+kappaz*z);
> y:=-(1/2)*a; z:=y; evalf(sigma);
> y:=(1/2)*a; z:=y; evalf(sigma);
```

2.2.1 $\sigma = \left(3\frac{Fl}{ta^3}\right)y + \left(-\frac{6}{7}\sqrt{3}\frac{Fl}{ta^3}\right)z$

Two randomly chosen points:

$y = z = -0.5a \quad \sigma = -0.758\frac{Fl}{ta^2}$

$y = z = 0.5a \quad \sigma = +0.758\frac{Fl}{ta^2}$

```
> restart;
units N, mm:
> zNC:=(40*4*2+56*4*(4+28))/(40*4+56*4);
> Iyy:=(1/12)*4*40^3+(1/12)*56*4^3;
> Izz:=(1/12)*40*4^3+(40*4*(zNC-2)^2)+(1/12)*4*56^3+4*56*(28+4-zNC)^2;
> M:=100e3: My:=-(1/5)*sqrt(5)*M: Mz:=-(2/5)*sqrt(5)*M;
Note : Sign is lost but load situation is given, so My and Mz are both negative!
> kappay:=My/(E*Iyy); kappaz:=Mz/(E*Izz);
> evalf(kappay/kappaz);
> alphak:=evalf( (180/Pi)*arctan(kappaz/kappay));
> alphaNA:=alphak-90;
> sigma:=simplify(evalf(E*(kappay*y+kappaz*z)));
> y:=0: z:='z': plot(sigma,z=-zNC..(60-zNC));
> z:=0: y:='y': plot(sigma,y=-20..20);
```

2.2.2 a) units N, mm
$\sigma = -2.06737y - 0.626560z$
b) *n.a.*, $\sigma = 0$, $z = -3.29956y$, $\alpha = -73.1°$
c) $\alpha_k = 16.9°$

2.2.3 A) (left cross section)
a) $\tan(\alpha_m) = 4/3$, $\alpha_m = 53°$ (explanation available at the Student Assistants)
b) Using Mohr's Circle method:
$I_{zz} = 3.413 \times 10^6$ mm^4
$I_{yy} = 5.973 \times 10^6$ mm^4
$I_{yz} = I_{zy} = 2.56 \times 10^6$ mm^4
$\alpha_1 = 31.7°$
$\alpha_2 = 301.7°$
c) $\tan(\alpha_m) = 4/3$, $\alpha_m = 53°$

B) (right cross section)
a) $\tan(\alpha_m) = -16/9$, $\alpha_m = -60.6°$
b) Using Mohr's Circle method:
$I_{zz} = 3.413 \times 10^6$ mm^4
$I_{yy} = 5.333 \times 10^6$ mm^4
$I_{yz} = I_{zy} = -1.92 \times 10^6$ mm^4
$\alpha_1 = -31.7°$
$\alpha_2 = 238.3°$
c) $\tan(\alpha_m) = -16/9$, $\alpha_m = -60.6°$

2.2.4 $\tan(\alpha_m) = -h/b$ (This is the other diagonal!!!)

2.2.5 This exercise is easier if you start with b)!
b) $\sigma = 0.5y$, at B: $\sigma = -20$ MPa
a) $M = 509$ Nm, $\alpha_m = -45°$ (force line through centre of gravity and B!)

2.2.6 $F_z = 14.62 \times 10^3 - 5.73F_y$ (N)
$F_{z,max} = 2.17$ kN
$F_{y,max} = 2.17$ kN

2.2.7 $I_{zz} = 4.93 \times 10^9$ mm^4
$I_{yy} = 1.733 \times 10^9$ mm^4
$I_{yz} = I_{zy} = 1.2 \times 10^9$ mm^4

Possibilities:
- $\sigma_A = +70$ MPa $\sigma_F = -50$ MPa $M_y = 240$ KNm $M_z = 987$ KNm
- $\sigma_A = -50$ MPa $\sigma_E = +70$ MPa try it yourself
- $\sigma_D = +70$ MPa $\sigma_F = -50$ MPa try it yourself

2.2.8 a) $\sigma_A = +18\sigma$ $\sigma_C = -6\sigma$
b) n.l. $2y + z = 0$
c) I_{yy} perpendicular to n.l. $= 7ta^3$

2.2.9 a) $\sigma_P = +125$ MPa $\sigma_Q = -83,3$ MPa
b) Shear force centre is Q and torsional moment :
$$M_t = 500 \times 15 = 7500 \text{ Nmm} \quad \text{(clock wise)}$$
c) use pseudo load approach : $F_y^* = 468.75$ N; $F_z^* = 781.25$ N; results in
$$u_y = \frac{F_y^* l^3}{3EI_{yy}} = 3.472 \text{ mm}; \quad u_z = \frac{F_z^* l^3}{3EI_{zz}} = 5.787 \text{ mm}; \quad u = 6.75 \text{ mm}$$
d) neutral axis : $3y + 5z = 0$, point of intersection with vertical part QP:
$$R_M^{(a)} = \tfrac{1}{2} \times (60 - 9 - 15) \times 5 \times 125 = 11250 \text{ N}$$
$$\tau = \frac{R_M^{(a)} \times 500}{5 \times 500 \times 1000} = 2.25 \text{ N/mm}^2$$

Note : use τ if no sign is involved.

2.2.10 $N_{BC} = +592$ N $\sigma = +79$ MPa
maximum $\tau = 1.4$ MPa (thin walled)
$$M_t = \frac{29 \times 592 + 1000 \times 29}{1000} = 46,16 \text{ Nm} \quad \text{(clock wise)}$$

The complete calculation sheet for MAPLE is given on the next page.

```
> restart;
> L:=2000; a:=100; t:=10; Fz:=1000;
> Es:=2.1e5;
```
with respect to corner point in cross section, NOT thin walled:
```
> A:=a*t+(a-t)*t; EA:=Es*A;
> zNC:=((a*t*a/2)+(a-t)*t*(t/2))/A; zNC:=evalf(round(%));
> yNC:=(a*t*a/2+(a-t)*t*t/2)/A; yNC:=evalf(round(%));
> Izz:=(1/12)*t*a^3+a*t*((a/2)-zNC)^2+(1/12)*(a-t)*t^3+(a-
t)*t*(yNC-t/2)^2;
> Iyy:=Izz; evalf(%);
> Iyz:=a*t*(yNC-t/2)*(zNC-(a/2))+(a-t)*t*(-t-(a-t)/2+yNC)*(zNC-
t/2); evalf(%);
> EIyy:=Es*Iyy: EIyz:=Es*Iyz: EIzz:=Es*Izz:
> Fy_pseudo:=(EIzz*EIyy*Fy-EIyz*EIyy*Fz)/(EIyy*EIzz-EIyz^2);
> Fz_pseudo:=(-EIyz*EIzz*Fy+EIyy*EIzz*Fz)/(EIyy*EIzz-EIyz^2);
```
no displacement in y-direction, so Fy-pseudo MUST be zero!! -> solve actual Fy
```
> eq:=Fy_pseudo=0; Fy:=solve(eq,Fy);
> V:=sqrt(Fy^2+Fz^2);
> My:=-Fy*L; Mz:=-Fz*L; M:=sqrt(My^2+Mz^2);
> eps:=0;
> kappay:=(1/(EIyy*EIzz-EIyz^2))*(EIzz*My-EIyz*Mz);
> kappaz:=(1/(EIyy*EIzz-EIyz^2))*(-EIyz*My+EIyy*Mz);
> strain:=eps+kappay*y+kappaz*z:
```
stresses in outer fibres (top and bottom since n.a. runs horizontal through NC):
```
> y:=yNC; z:=-a+zNC; SigmaTop:=Es*strain;
> y:=yNC; z:=+zNC; SigmaBot=Es*strain;
```
displacements of Z:
```
> uy:=Fy_pseudo*L^3/(3*EIyy);
> uz:=Fz_pseudo*L^3/(3*EIzz);
> u:=sqrt(uy^2+uz^2);
```
shear stress (thin walled): (where neutral axis intersects with vertical part of the cross section)
```
> RM:=(1/2)*(a-zNC)*t*SigmaTop;
> tau:=RM*V/(t*M);
> Mt:=Fz*yNC-Fy*29;
```

Normal stresses due to bending and extension

2.3.1 a) At D σ_{xx} = +1 MPa At B σ_{xx} = -2 MPa (nl 3y = 100)
b) At D σ_{xx} = - 0.5 MPa At B σ_{xx} = -3.5 MPa (nl 3y + 6z - 100 = 0)

2.3.2 a) At C σ_{xx} = 1.818 MPa nl: y = 55

```
> restart;
assignment 2.3.2 a) : units N, mm:
> F:=10e3; t:=10;
> A:=2*150*t+200*t;
> yNC:=(2*150*t*75+200*t*150)/A;
> Iyy:=200*t*(yNC-150)^2+2*((1/12)*150^3*t+150*t*(yNC-75)^2);
> Iyz:=0;
> Izz:=(1/12)*t*200^3+2*150*t*100^2; evalf(%);
deformation of the cross section:
> eps:=N/(Emod*A);
> kappay:=1/(Emod*Iyy*Emod*Izz-Emod*Iyz*Emod*Iyz)*
          (Emod*Izz*My-Emod*Iyz*Mz);
> kappaz:=1/(Emod*Iyy*Emod*Izz-Emod*Iyz*Emod*Iyz)*
          (-Emod*Iyz*My+Emod*Iyy*Mz);
> sigma:=Emod*(eps+kappay*y+kappaz*z);
a) loading at A:
> N:=-F; ey:=-45; ez:=0; My:=N*ey; Mz:=N*ez;
> simplify(27.5*sigma);
sigma at C:
> y:=105; z:=100; evalf(sigma);
sigma at B:
> y:=-45; z:=-100; evalf(sigma);
sigma at A:
> y:=-45; z:0; evalf(sigma);
```

b) At C σ_{xx} = 4.545 MPa nl: 4y + 3z –220 = 0

2.3.3 A) (left cross section)
a) $N = 1/4 \times hb\sigma$
b) $y_k = 0$ $z_k = -1/2 \times h$

B) (right cross section)
a) $N = 2\sigma a^2$
b) $y_k = 1/4a$ $z_k = -1/3a$

Inhomogeneous cross sections loaded in extension

2.4.1 a) σ_s = - 8.58 MPa σ_b = - 0.60 MPa
b) ΔL = 0.17 mm
c) N_{max} = 2660 kN (concrete is governing)

2.4.2 a) N = 2188 kN (concrete is governing)
b) NC vertically in the middle, horizontally at 336.5 mm starting at the left side.

2.4.3 a) For a coordinate system with its origin at the top right corner:
y-axis pointing to the left, z-axis pointing down:
centre of gravity: y = 36.41 mm z = 50.77 mm
b) F = 312 kN, material 4 is governing

Inhomogeneous cross sections loaded in bending

2.5.1 a) F = 672 N
b) F = 3808 N
c) F = 1232 N
d) F = 6203 N

2.5.2 Starting at the top (N/mm^2):
In 1 starting at 60 to 24 (straight line)
In 2 starting at 32 to -16
In 3 starting at –20 to –80

(You will have a jump in the stress distribution)

2.5.3 $\kappa_z = 0.212$ m^{-1}
$\varepsilon^T = 2.55 \times 10^{-3}$

Stress distribution starting at the top (N/mm^2):
In 1 starting at +33.6 to –51.2
In 2 starting at +23.05 to – 5.36

(You will have a jump in the stress distribution)

2.5.4 a) F = -226 kN
b) e = 438 mm from the left side; (82 mm left of the right side)
e_y = - 127 mm (from NC of cracked cross section)

2.5.5 The position of the NC is 271 mm from the top.
Stress at the top is -2.83 N/mm^2
at the bottom is 1.34 N/mm^2
in the rebar 12.34 N/mm^2

2.5.6 The position of the NC is 258 mm from the top.
Stress at the top is -2.94 N/mm^2
in the rebar 15.77 N/mm^2

```
> restart;
assignment 2.5.5 and 2.5.6
units N, mm:
> As:=4800; Mz:=20e6;
> Ec:=14000; Es:=210000;
> b:=200; h:=400; a:=50;
NC for uncracked cross section with respect to top: (assume As << Ac)
> EA:=As*Es+(b*h)*Ec:
> zNC:=(Es*As*(h-a)+Ec*b*h*(h/2))/EA: evalf(zNC);
Bending stiffness:
> EIzz:=Ec*((1/12)*b*h^3+b*h*(zNC-h/2)^2)+Es*As*(zNC-(h-a))^2:
Stresses in concrete and steel:
> SIGc_top:=evalf(Ec*(Mz*(-zNC)/EIzz));
> SIGc_bot:=evalf(Ec*(Mz*(h-zNC)/EIzz));
> SIGs:=evalf(Es*(Mz*(h-a-zNC)/EIzz));

NC for cracked cross section with respect to top:
> EA:=As*Es+b*x*Ec:
> eq:=((Es*As*(h-a)+Ec*b*x*(x/2))/EA)=x:
> sol:=solve(eq,x): x:=sol[2]: evalf(x);
Bending stiffness:
> EIzz:=Ec*(1/3)*b*x^3+Es*As*(h-a-x)^2:
Stresses in concrete and steel:
> SIGc_top:=evalf(Ec*(Mz*(-x)/EIzz));
> SIGs:=evalf(Es*(Mz*(h-a-x)/EIzz));
>
```

Core

2.6.1 a) + b) The core will be a triangle with the corner points with respect to the centre of gravity:

1) $e_y = -1/12 \times a$
$e_z = 1/12 \times a$
2) $e_y = -1/12 \times a$
$e_z = -1/6 \times a$
3) $e_y = +1/6 \times a$
$e_z = +1/12 * a$

c) $\sigma_A = 24 \times M/a^3$
$\sigma_B = -24 \times M/a^3$
$\sigma_C = 0$ (at the neutral line)

d) force line with the angle: +27° degrees with respect to the y-axis, through C.

2.6.2 a) Core will be a quadrilateral with the corner points with respect to the NC:

1) $e_y = 0$
$e_z = +7/9 \times a$
2) $e_y = 0$
$e_z = -7/9 \times a$
3) $e_y = +1/9 \times a$
$e_z = 0$
4) $e_y = -1/9 \times a$
$e_z = 0$

b) Core will be a quadrilateral with the corner points with respect to the NC:

1) $e_y = 0$
$e_z = +13/15 \times a$
2) $e_y = 0$
$e_z = -13/15 \times a$
3) $e_y = +4/15 \times a$
$e_z = 0$
4) $e_y = -4/15 \times a$
$e_z = 0$

c) Core will be a quadrilateral with the corner points with respect to the NC:

$I_{yy} = \frac{7}{4} ta^3; I_{yz} = -ta^3; I_{zz} = \frac{52}{15} ta^3;$

1) ey = - 5/22×a 2) ey = - 1/4×a
ez = - 2/15×a ez = + 13/15×a
3) ey = + 7/10×a 4) ey = + 1/6×a
ez = - 2/5×a ez = -26/45×a

2.6.3 A) (left cross section)

a) $I_{yy} = 48 \cdot ta^3$ and $I_{zz} = 85/3 \cdot ta^3$

Core will be a triangle with the corner points with respect to the NC:

1) $e_y = 0$
   $e_z = + 17/5 \times a$
2) $e_y = + 16/11 \times a$
   $e_z = - 85/132 \times a$
3) $e_y = - 16/11 \times a$
   $e_z = -85/132 \times a$

b) stresses in the corner points:

$\sigma_A = -83/272 \cdot F/ta$ (with F taken as a compressive force at A)

$\sigma_B = 19/272 \cdot F/ta$

$\sigma_C = 1/17 \cdot F/ta$

```
> restart;
assignment 2.6.3 a)
units N, mm:
(c) 2021 : Hans Welleman
=================================================================
method 1 : massive approach, outer triangle 1 and inner triangle 2
> t:=0.00001*a; # assume thin walled ....
> h1:=4*a; b1:=6*a; h2:=h1-t-(5/3)*t; b2:=b1-2*((5/4)+(3/4))*t;
> A1:=(1/2)*h1*b1; A2:=(1/2)*h2*b2;
> A:=A1-A2;
> zNC:=(A1*(1/3)*h1-A2*((1/3)*h2+t))/A; simplify(%);
> Izz:=(1/36)*b1*h1^3+A1*(zNC-(h1/3))^2-((1/36)*b2*h2^3+A2*(zNC-t-(h2/3))^2);
> Iyy:=(1/48)*b1^3*h1-(1/48)*b2^3*h2; %/t;
> Iyz:=0; # by definition due to symmetry
method 2 : thin walled approach based on strips (see Hartsuijker & Welleman
volume 2, page 131-132):
> restart;
> A:=6*a*t+10*a*t;
> zNC:=2*5*a*t*2*a/A;
> Izz:=6*a*t*zNC^2+2*((1/12)*(5*t/4)*(4*a)^3+5*a*t*(zNC-(2*a))^2);
> Iyy:=(1/12)*t*(6*a)^3+2*((1/12)*(5*t/3)*(3*a)^3+5*a*t*((3*a/2))^2);
> Iyz:=0; # by definition due to symmetry
> ey:=-(1/A)*(Iyy*(1/y1)+Iyz*(1/z1));
> ez:=-(1/A)*(Iyz*(1/y1)+Izz*(1/z1));
neutral axis at top:
> y1:=infinity; z1:=(-zNC); ey1:=simplify(ey); ez1:=simplify(ez);
> evalf(ey); evalf(ez);
neutral axis along left side:
> y1:=(33*a)/16; z1:=(11*a)/4; ey1:=simplify(ey); ez1:=simplify(ez);
> evalf(ey); evalf(ez);
neutral axis along right side:
> y1:=(-33*a)/16; z1:=(11*a)/4; ey1:=simplify(ey); ez1:=simplify(ez);
> evalf(ey); evalf(ez);
```

B) (right cross section)

a) $I_{yy} = 21 \cdot ta^3$ en $I_{zz} = 19/2 \cdot ta^3$

Core will be a triangle with the corner points with respect to the NC:

1) e_y = - 1/4×a
   e_z = - 19/24×a
2) e_y = + 7/6×a
   e_z = + 1/6×a
3) e_y = - 3×a
   e_z = + 13/12×a

b) Do it yourself.

2.6.4 a) n.l.: z = y + 350 (in mm)
e_y = + 21.4 mm
e_z = - 38 mm

b) stresses in the corner points:

σ_A = -0.71 MPa

σ_B = 0

σ_C = -0.95 MPa

σ_D = -1.66 MPa

2.6.5 Core will be a hexagon with the corner points with respect to the NC:

1) e_y = - 33.33 mm
   e_z = 0
2) e_y = - 94.1 mm
   e_z = - 20.43 mm
3) e_y = 0
   e_z = + 64.5 mm
4) e_y = + 33.33 mm
   e_z = 0
5) e_y = + 94.1 mm
   e_z = - 20.4 mm
6) e_y = 0
   e_z = - 49.3 mm

The bending moment will cause tensile stresses at the bottom. The force point of the reinforcement should be positioned below core point 6. If the reinforcement (without the bending moment) is positioned in core point 3, the neutral line will be in the upper fibre.
With bending moment, the reinforcement should be positioned below core point 3 to resist the bending moment in the best way.

2.6.6 a) positioning with respect to D:
y_{NC} = 50 mm
z_{NC} = 26.76 mm
Moments of inertia:

I_{zz} = 780,000 mm^4, I_{yy} = 945,000 mm^4, I_{yz} = - 585,000 mm^4
Core will be a quadrilateral with the corner points with respect to the NC:
1) e_y = + 6.5 mm
   e_z = - 8.67 mm
2) e_y = + 10.36 mm
   e_z = - 3.10 mm
3) e_y = -8.13 mm
   e_z = + 10.83 mm
4) e_y = - 5.71 mm
   e_z = -3.10 mm

b) $\sigma(y,z) = +222.2y + 359z$, in N/mm^2
Stresses in the corner points:
σ_A = + 9.75 x 10^3 N/mm^2
σ_B = + 3.08 x 10^3 N/mm^2
σ_C = - 11.80 x 10^3 N/mm^2
σ_D = + 1.54 x 10^3 N/mm^2

c) $\kappa_y = 0$, $M_y / M_z = -1.077$
angle with respect to the horizontal line: 43° (clock wise)

2.6.7 A) (right cross section)

a) Positioning of NC with respect to A:
y_{NC} = -25 mm
z_{NC} = + 11.67 mm
Moments of inertia:
I_{zz} = 107,500 mm^4, I_{yy} = 227,500 mm^4, I_{yz} = 60,000 mm^4
Stress formula: $\sigma(y,z) = -2.8768y + 10.9080z$, in N/mm^2
Stresses in the corner points:
σ_A = - 199 N/mm^2
σ_B = - 27 N/mm^2
σ_C = + 83 N/mm^2
σ_D = - 33 N/mm^2
σ_E = + 295 N/mm^2
σ_F = + 266 N/mm^2
σ_G = - 61 N/mm^2
σ_H = - 90 N/mm^2

b) condition: $\sigma_A = -\sigma_E$, which causes: $\kappa_y = -5/9 \cdot \kappa_z$
angle with respect to the horizontal line: 48° (clock wise)

c) $\kappa_y = 0$
Rotate the profile to the principal direction:
angle with respect to the horizontal line: 29° (anti clock wise)

B) (Left cross section)

a) Positioning of NC with respect to A:
y_{NC} = - 6.67 mm
z_{NC} = + 13.33 mm
Moments of inertia:
I_{zz} = 35,556 mm^4, I_{yy} = 8,389 mm^4, I_{yz} = 8,389 mm^4
Stress formula: $\sigma(y,z) = -36.809y + 36.809z$, in N/mm^2
Stresses in the corner points:
σ_A = - 736 N/mm^2
σ_B = 0 N/mm^2
σ_C = + 736 N/mm^2

b) Condition: $\sigma_A = -\sigma_B$, which causes: $\kappa_y = -4 \cdot \kappa_z$
angle with respect to the horizontal line: 903° (clock wise)

c) $\kappa_y = 0$
Rotate the profile to the principal direction:
angle with respect to the horizontal line: 14° (anti clock wise)

2.6.8 a) Positioning of NC with respect to D:
y_{NC} = - 100 mm
z_{NC} = + 50 mm
Moments of inertia:
I_{zz} = 585 x 10^6 mm^4, I_{yy} = 270 x 10^6 mm^4, I_{yz} = - 180 x 10^6 mm^4

b) Core will be a quadrilateral with the corner points with respect to the NC:
1) e_y = - 150 mm
e_z = + 100 mm
2) e_y = + 20 mm
e_z = + 90 mm
3) e_y = + 75 mm
e_z = - 50 mm
4) e_y = + 40 mm
e_z = - 130 mm

c) stress formula: $\sigma(y,z) = 0.11 - 0.0011y$, in N/mm^2
σ_A = 0
σ_B = + 0.33 N/mm^2
σ_C = + 0.33 N/mm^2
σ_D = 0
σ_E = 0

Shear stresses due to bending

2.7.1 $I_{zz} = \frac{2}{3} t\sqrt{2} \cdot a^3$

$\tau_A = 0$

$\tau_B = \dfrac{3Q}{4at}$

$\tau_C = 0$

Line of action: 0.5×a left with respect to NC ($y = +0.5 \times a$)

2.7.2 $I_{zz} = \frac{5}{24} ta^3$, $I_{yy} = \frac{5}{24} ta^3$, $I_{yz} = \frac{1}{8} ta^3$

Neutral line: $-3y + 5z = 0$

Normal stresses in the corner points:

$\sigma_A = \dfrac{9M}{2ta^2}$

$\sigma_B = -3\dfrac{M}{ta^2}$

$\sigma_C = \dfrac{3M}{2ta^2}$

Shear stresses in the corner points:

$\tau_A = \tau_C = 0$

$\tau_B = \dfrac{3Q}{4at}$

Maximum shear stress in AB: $\tau_{AB,\max} = 1.35 \cdot \dfrac{Q}{at}$

Maximum shear stress in BC: $\tau_{BC,\max} = \dfrac{Q}{4at}$

Line of action: 0.25×a right with respect to NC ($y = -0.25 \times a$)

2.7.3 shear stresses in A,D,B,C,E,H:

$\tau_A = \tau_D = \dfrac{Q}{4th}$

$\tau_B = \tau_C = \dfrac{11Q}{12th}$

$\tau_E = 0$

$\tau_H = \dfrac{7Q}{6th}$

Line of action: $y = -\frac{7}{9} h$

2.7.4 Moments of inertia with respect to NC:

$I_{zz} = \frac{37}{48} ta^3$

$I_{yy} = 2\tfrac{7}{16} ta^3$

$I_{yz} = -1\tfrac{1}{16} ta^3$

Neutral line: $125y + 219z = 0$

Normal stresses in the corner points:

$$\sigma_A = 3\frac{13}{36}\frac{M}{ta^2}$$

$$\sigma_B = -3\frac{7}{12}\frac{M}{ta^2}$$

$$\sigma_C = 2\frac{1}{2}\frac{M}{ta^2}$$

$$\sigma_D = -\frac{35}{36}\frac{M}{ta^2}$$

Shear stresses in the corner points:

$$\tau_A = \tau_D = 0$$

$$\tau_B = -0.22\frac{M}{\ell ta}$$

$$\tau_C = -0.764\frac{M}{\ell ta}$$

Maximum shear stress in AB: $\tau_{AB,\max} = 1.627 \cdot \dfrac{M}{\ell ta}$

Maximum shear stress in BC: $\tau_{BC,\max} = -1.278\dfrac{M}{\ell ta}$

Maximum shear stress in CD: $\tau_{CD,\max} = 0.136\dfrac{M}{\ell ta}$